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PREDICTION OF HEAT TRANSFER BY NATURAL CONVECTION IN CLOSED CYLINDERS HEATED FROM BELOW

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Abstract—Use is made of the Malkus–Veronis power integral technique to predict heat-transfer rates in fluids contained in vertical right cylinders heated on a horizontal bottom and cooled on a horizontal top end. Results are obtained for cells with either thermally insulating or conducting sidewalls. Closed-form relations are given for Nusselt number in terms of Rayleigh number based on cell height and on an "adjusted wave-number" which depends on height-to-diameter ratio for circular cylinders and height-to-side ratios for square cylinders. Comparisons between the predictions and previously reported data are shown.

NOMENCLATURE

a, horizontal wavenumber;
A_f, fluid cross-sectional area;
A_w, wall cross-sectional area;
b, vertical wavenumber;
C_p, specific heat at constant pressure;
d, cell diameter;
f_{mn}, function of x and y;
F_k, function of x and y;
g, gravitational acceleration;
$g(z)$, function of z;
H, side of a square or width of a slot;
k, z-mode number;
K, thermal conductivity;
K_w, wall conductivity;
L, cell height;
m, horizontal mode number;
M, highest z-mode of convection initiated;
n, horizontal mode number;
N, power integral coefficient, equation (23);
Nu, Nusselt number (K_{Eff}/K);
q, heat flux;
$\dot{Q}$, heat rate per cell;
$\mathcal{R}$, Rayleigh number ($\alpha g \beta L^4/\kappa\nu$);
t, time;
T, temperature;
T', temperature disturbance;
T_c, top cold temperature;
T_h, bottom hot temperature;
v, velocity vector;
w, z-velocity component;
w', disturbance in w;
x, horizontal coordinate;
y, horizontal coordinate;
z, vertical coordinate.

Greek symbols

α, volume thermal expansion coefficient;
β, temperature gradient in negative z-direction;
θ, dimensionless temperature;
κ, thermal diffusivity;
ν, kinematic viscosity;
π, 3.1415 . . . ;
ρ, fluid density;
Ω, dimensionless z-velocity.

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INTRODUCTION

ANALYTICAL prediction of heat transfer through a fluid contained between infinite parallel horizontal plates heated from below has been made by Malkus and Veronis [1] using an integral technique applied by Stuart [2] to the mathematically analogous problem of predicting momentum transfer through a fluid between two closely-spaced, concentric, rotating cylinders. Stuart treated the contribution of the first mode of convection, and Malkus and Veronis calculated contributions by the first five modes of convection.

The phrase "mode of convection" was explained by Stuart as follows in regard to laminar channel flow: "... the occurrence of instability in a flow may lead to the replacement of the original laminar flow by a new laminar flow, which consists of a mean flow with a superimposed finite disturbance. The flow may be expected to persist for a certain range of Reynolds number above the critical value and then to become unstable at some Reynolds number against a new (second) type of disturbance. A new equilibrium flow, consisting of a mean flow with two superimposed modes of disturbance, is then conceivable for a range of Reynolds number above the second critical value. As the Reynolds number is raised still further, additional modes of disturbance may appear successively until, at a sufficiently large Reynolds number, the flow is so highly disturbed as to be considered turbulent." Stuart went on to point out that this word picture would apply as well to natural convection between horizontal plates, when the "original laminar flow" is still fluid and "Reynolds number" is replaced by Rayleigh number. The critical Rayleigh number at which a new mode of convection initiates was taken by Malkus and Veronis from the linear stability theory results of Pellew and Southwell [3].

Nakagawa [4] attempted to use power integrals as outlined by Malkus and Veronis and by Stuart. To do so, he considered an infinite horizontal layer of liquid with two rigid boundaries, one rigid and one free, and two free boundaries. He obtained expressions for the amplitudes of the disturbances for the first mode and then a general expression for the Nusselt number which corresponds with that of Malkus and Veronis, as it should. In calculating the power integral coefficient, he obtained the correct results for the free surfaces case, $N_1 = 2$, but obtained an erroneous result 50 per cent low for the rigid surface case. No comparison with experiment was made. Hollands [5] recomputed the values of N_k for the first two modes. His results agreed within 10 per cent of those of Malkus and Veronis and compared well with experiment. Catton [6] computed N_k for the first ten modes and used the N_k to calculate the heat transfer for $10^3 < R < 10^7$. He found excellent agreement with Silveston's [7] experimental data for infinite horizontal surfaces.

The presence of lateral walls between the horizontal plates raises the Rayleigh number at which a mode of convection initiates, if the walls constrict the convective disturbance [8–10]. As a first approximation, it might be expected that the results of Malkus and Veronis could be applied to this constricted convection, making allowance for the change in critical Rayleigh numbers. For this reason, the Malkus–Veronis power integral technique is examined for application to natural convection inside walled cells. Such a procedure will result in relations containing no free parameters to be empirically determined by fitting experiments. Such empirical correlations, often quite useful, will not be considered in what follows. The theoretical results to be obtained will be compared with the experimental measurements reported previously [11].

HEAT-TRANSFER ANALYSIS

Fundamentals

It is desired to estimate the natural convection heat transfer upward through a fluid inside a right cylinder, not necessarily circular in cross-section. The lower end wall is heated to maintain it at temperature T_h, and the top end is cooled to temperature T_c. Two types of sidewalls are considered, one a good conductor, so that the wall temperature is linear in the upward direction despite heat exchange with the fluid in the cylinder, and the other is a poor conductor and radiator so that negligible heat is exchanged between the fluid and sidewall. In both cases, radiation exchange within the fluid and other heat sources or sinks within the fluid are neglected.

For a given cylindrical cell and fluid, if the temperature difference $T_h - T_c$ is sufficient, natural convection will occur, warm light fluid rising in some regions and cold heavy fluid falling in other regions under the influence of a downward-directed gravitational-body-force field. As a result of the fluid motion, warm fluid will be carried to the vicinity of the cold end wall and cold fluid to the vicinity of the hot end wall, thus increasing thermal conduction at these walls.

The Malkus power integral technique can be adopted to estimate the heat transfer in cells heated from below. It utilizes an integral expression derived from the equation of conservation of energy. Into this expression are substituted approximate velocity and temperature profiles for the most unstable convective disturbances, which are derived from a linear perturbation analysis of the equations of motion.

At any position in the fluid or sidewall, the upward heat flux is composed of a conduction term and a convective one

$$q = -K\frac{\partial T}{\partial z} + \rho C_p w T. \qquad (1)$$

The average upward heat rate $\dot{Q}$ for the entire cell volume, including the wall and neglecting property variation, is

$$\dot{Q} = \langle \overline{q_w} \rangle A_w + \langle \bar{q} \rangle A_f = A_w K_w \langle \overline{\beta_w} \rangle + A_f \langle \bar{\beta} \rangle + A_f \rho C_p \langle \overline{wT} \rangle, \qquad (2)$$

where the bar denotes a horizontal average, the caret $\langle\,\rangle$ denotes a vertical average, β denotes $-\mathrm{d}T/\mathrm{d}z$, A_w is the cross-sectional wall area, and A_f is the fluid cross-sectional area. We restrict the temperature profiles to be horizontally symmetrical (odd) in such a way that no fluid volume element $A_f\,\mathrm{d}z$ has a net transfer of heat with a wall volume element $A_w\,\mathrm{d}z$. This restriction is consistent with the profiles of the most unstable disturbances [12]. With this restriction, the horizontal average upward heat flux in the fluid is a constant

$$\bar{q} = K\bar{\beta} + \rho C_p \overline{wT} = \langle \bar{q} \rangle \qquad (3)$$

and the volume average heat flux can be separated out of equation (2)

$$\langle \bar{q} \rangle = K\langle \bar{\beta} \rangle + \rho C_p \langle \overline{wT} \rangle. \qquad (4)$$

The temperature profiles to be used are likewise restricted to be vertically symmetrical (odd) so that

$$\langle \bar{\beta} \rangle = \frac{T_h - T_c}{L}, \qquad (5)$$

where L is the height of the cell. Equations (4) and (5) may be combined to form a Nusselt number

$$Nu = \frac{\bar{q}L}{K(T_h - T_c)} = 1 + \frac{\langle \overline{wT} \rangle}{\kappa \langle \bar{\beta} \rangle}, \qquad (6)$$

where κ is the thermal diffusivity $K/\rho C_p$.

Velocity and temperature profiles

Equation (6) is not useful by itself, because a velocity and a temperature profile must be chosen and the amplitudes determined. The profiles will be taken to be those obtained from a linear perturbation analysis of the thermal stability problem. This approximation is sometimes called the shape assumption [2]. It is the properties of these profiles that permit $\langle \overline{wT} \rangle / \kappa \langle \bar{\beta} \rangle$ in equation (6) to be expressed simply as a sum of terms containing only ratios of volume averages and ratios of Rayleigh numbers so that the amplitudes of the velocity and temperature profiles will not enter directly.

With no loss in generality, the temperature can be expressed as a sum of a z-dependent term and a temperature perturbation

$$T(x, y, z) = \overline{T}(z) + T'(x, y, z). \quad (7)$$

Since conservation of fluid requires

$$\overline{w} = 0, \quad (8)$$

there holds

$$\overline{wT} = \overline{wT'}. \quad (9)$$

Solutions to the linearized conservation equations, mass, momentum and energy can be found when either horizontal harmonicity ($L/d \to 0$) or vertical harmonicity ($L/d \to \infty$) could be taken. In either case, the velocity is expressed according to

$$w = w' = \sum_k \sum_m \sum_n f_{k,m,n}(x, y) g_k(z). \quad (10)$$

When vertical harmonicity is assumed and boundary conditions $w = 0$ at $z = \pm L/2$ were taken, $g_k(z)$ is given by a simple harmonic function in phase with w_k, e.g. for odd k

$$w = \sum_k [\sum_m \sum_n f_{k,m,n}(x, y)] \cos(k\pi z/L),$$

$$T' = \sum_k [\sum_m \sum_n F_{k,m,n}(x, y)] \cos(k\pi z/L).$$

It follows from this relationship between w_k and T'_k that

$$\langle \overline{wT'} \rangle = \sum_k \langle \overline{w_k T'_k} \rangle. \quad (11)$$

Equation (11) remains valid when horizontal harmonicity is employed in place of vertical harmonicity, by virtue of the harmonicity in the horizontal plane.

In applying the integral technique, it is necessary to fix arbitrarily upon a definite profile for the velocity and temperature disturbances,

$$f_k(x, y) = \sum_m \sum_n f_{k,m,n}(x, y),$$

$$F_k(x, y) = \sum_m \sum_n F_{k,m,n}(x, y).$$

For this purpose, the one existing at the onset of convection is chosen, that is, the one prevailing at the lowest value of Rayleigh number $\mathscr{R}_k$ at which the kth mode of convection initiates.

The Malkus power integral

Equation (6) must be expressed in terms of ratios of velocities and temperatures. To accomplish this transformation, the complete energy equation and the properties of the arbitrarily selected profiles are used. The energy equation is written in terms of T' for steady flow by introduction of Equation (7) and $d/dt = 0$. Multiplied by T'_k and integrated over the cell volume it gives

$$\kappa \langle \overline{T'_k \nabla^2 T'} \rangle = - \langle \overline{\beta} (\overline{wT'_k}) \rangle + \langle \overline{T'_k v \,.\, \nabla T'} \rangle + \kappa \left\langle \overline{T'_k} \frac{\partial \overline{\beta}}{\partial z} \right\rangle. \quad (12)$$

In this integral energy equation, temperature profiles will be taken in the form mentioned

$$T' = \sum_k T'_k; \quad (13)$$

$$\overline{\beta} = \langle \overline{\beta} \rangle + \sum_k \overline{\beta}_k. \quad (14)$$

Subtraction of equation (4) from (3) and introduction of equations (11, 13, 14) yields a relation required by conservation of energy

$$\kappa \sum_k \overline{\beta}_k = \sum_k \{ \langle \overline{w_k T'_k} \rangle - \overline{w_k T'_k} \}. \quad (15)$$

Differentiation of equation (3) likewise yields

$$\frac{\partial \overline{\beta}}{\partial z} = \frac{1}{\kappa} \frac{\partial}{\partial z} (\overline{wT}) = \frac{1}{\kappa} \sum_k \frac{\partial}{\partial z} (\overline{wT'_k}). \quad (16)$$

Equations (12) and (15) are satisfied by taking them to hold term by term. The triple-product term in equation (12) disappears by virtue of the harmonicity in either the horizontal or vertical directions. Equation (12) becomes

$$\kappa \langle \overline{T'_k \nabla^2 T'_k} \rangle = - \langle \overline{\beta} \rangle \langle \overline{w_k T'_k} \rangle - \langle \overline{w_k T'_k} \rangle^2 \kappa^{-1} + \langle (\overline{w_k T'_k})^2 \rangle \kappa^{-1}. \quad (17)$$

This relation may be divided by $\langle \overline{w_k T'_k} \rangle$ and the result written to express $\langle \overline{w_k T'_k} \rangle$ in the desired terms of ratios,

$$\frac{\langle \overline{w_k T'_k} \rangle}{\kappa \langle \overline{\beta} \rangle} = \frac{1 + \dfrac{\kappa \langle \overline{T'_k \nabla^2 T'_k} \rangle}{\langle \overline{\beta} \rangle \langle \overline{w_k T'_k} \rangle}}{\dfrac{\langle \overline{(w_k T'_k)^2} \rangle}{\langle \overline{w_k T'_k} \rangle^2} - 1}. \tag{18}$$

Dimensionless profiles are introduced by scaling all lengths to L and taking Ω and θ as follows:

$$L = 1, \quad \Omega_k = \frac{L}{\kappa} w_k, \quad \theta_k = \frac{\alpha g L^3}{\kappa \nu} T'_k, \tag{19}$$

equation (18) becomes

$$\frac{\langle \overline{w_k T'_k} \rangle}{\kappa \langle \overline{\beta} \rangle} = \frac{1 + \dfrac{1}{\mathscr{R}} \dfrac{\langle \overline{\theta_k \nabla^2 \theta_k} \rangle}{\langle \overline{\Omega_k \theta_k} \rangle}}{\dfrac{\langle \overline{(\Omega_k \theta_k)^2} \rangle}{\langle \overline{\Omega_k \theta_k} \rangle^2} - 1}, \tag{20}$$

where $\mathscr{R}$ is the Rayleigh number.

The profiles to be introduced in the volume integrals are those given by the linearized perturbation equations, i.e. the shape assumption is employed. The linearized energy equation indicates

$$-w_k \langle \overline{\beta}_k \rangle = \kappa \nabla^2 T'_k;$$
$$-\mathscr{R}_k \Omega_k = \nabla^2 \theta_k. \tag{21}$$

This relation permits equation (18) to be written in a convenient form

$$\frac{\langle \overline{w_k T'_k} \rangle}{\kappa \langle \overline{\beta} \rangle} = N_k \left(1 - \frac{\mathscr{R}_k}{\mathscr{R}}\right), \mathscr{R} > \mathscr{R}_k, \tag{22}$$

where

$$N_k = \frac{1}{\dfrac{\langle \overline{(\Omega_k \theta_k)^2} \rangle}{\langle \overline{\Omega_k \theta_k} \rangle^2} - 1}. \tag{23}$$

Equations (11) and (12) can now be introduced into equation (6) to obtain the useful result.

$$Nu = 1 + \sum_{k=1}^{M} N_k \left[1 - \frac{\mathscr{R}_k}{\mathscr{R}}\right], \tag{24}$$

where M is the highest mode of convection initiated for a given Rayleigh number,

$$\mathscr{R}_{k=M} < \mathscr{R} \leqslant \mathscr{R}_{k=M+1}. \tag{25}$$

Values of $\mathscr{R}_k$, N_k, and Nu

Values of $\mathscr{R}_k$ can be determined for very low L/d as was done by Pellew and Southwell [3] or for very high L/d as was done by Yih [9]. At intermediate values it is possible to estimate the effects of horizontal shear on the top and bottom ends by adding a correction to the vertical wavenumber b_k used in the free surface expression [13]. When this correction is made the resulting formalism for estimating $\mathscr{R}_k$ results.

1. For a particular horizontal cross-section obtain the horizontal wavenumber, a, in the manner of Ostrach and Pnueli [10] by solving

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -a^2 w \tag{26}$$

subject to

$$w = 0 \text{ on the lateral walls} \tag{27}$$

and

$$\int_{A_f} w \, dA_f = 0. \tag{28}$$

There results for a square cylinder

$$a = \frac{\sqrt{(5)}\, \pi L}{H} \tag{29}$$

and for a circular cylinder of diameter d

$$a = \frac{7 \cdot 66 L}{d}. \tag{30}$$

2. Account for wall conduction by taking an adjusted wavenumber a_0,

$$a_0 = 0 \cdot 75 a \tag{31}$$

for adiabatic sidewalls or,

$$a_0 = a \tag{32}$$

for perfectly conducting sidewalls.

3. Account for the fact that the cellular convection is not much influenced by lateral walls at low L/d by taking

$$a_k^2 \doteq a_0^2 + (b_k^2/2). \tag{33}$$

4. Account for horizontal shear on the ends by taking for closed cells

$$b_k \doteq k\pi + 0 \cdot 85. \tag{34}$$

5. Find an approximate value of $\mathscr{R}_k$ from the relation

$$\mathscr{R}_k = \frac{(a_k^2 + b_k^2)^3}{a_k^2}. \quad (35)$$

Equations (33) and (35) written in the form shown are correct in the limit as L/d approaches infinity, for in this case b_k goes to zero leaving Rayleigh number equal to a_0^4. In the limit as L/d approaches zero they are likewise correct, for in this case the convection cells break loose from the sidewalls and assume a size such that a_k^2 is $b_k^2/2$. The formalism shown gives a useful procedure for intermediate cases, as will be shown by comparison of predictions with experimental results.

Values of N_i are determined from equation (23) using the Ω_k and θ_k determined by Catton [13]. Table 1 shows results for $L/d = 0$. At high values of horizontal wavenumber a the effects of the ends become small with the result that the vertical variations approach $\cos(k\pi z/L)$ for odd k or $\sin(k\pi z/L)$ for k even. In this case $N_k = 2$. As an approximation it is suggested that the values of $N_k(a)$ in Table 1 can be used for any L/d when the approximate value of a_i determined as above is used.

Calculations of Nusselt number vs. Rayleigh number for a circular cylinder with various L/d values and for adiabatic or perfectly conducting sidewalls were made using the values of $\mathscr{R}_k$ and N_k obtained. Figures 1 and 2 show the results. Experimental data taken from [11] are shown for comparison. Agreement within approximately 10 per cent is seen to exist.

DISCUSSION

An analysis based upon an integral technique presents difficulties when it is desired to assess

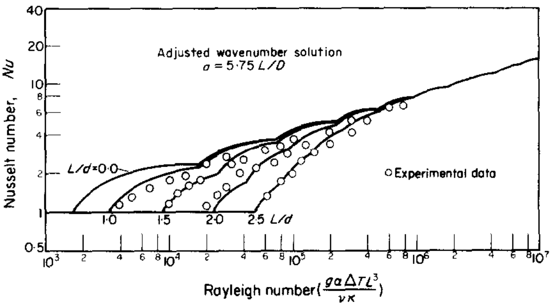

FIG. 1. Heat transfer in a cylinder with adiabatic walls.

Table 1. Power integral coefficient for infinite horizontal surfaces

Wave number	1	2	3	4	5	6	7	8	9	10
1·000	1·435	1·651	1·745	1·805	1·850	1·886	1·918	1·946	1·974	2·000
3·116	1·446	1·655	1·749	1·808	1·851	1·887	1·918	1·947	1·975	2·000
5·365	1·475	1·664	1·755	1·812	1·854	1·889	1·920	1·948	1·976	2·000
7·582	1·517	1·676	1·762	1·817	1·858	1·892	1·922	1·949	1·977	2·000
9·801	1·564	1·691	1·769	1·822	1·861	1·895	1·924	1·951	1·979	2·000
12·020	1·609	1·707	1·777	1·827	1·865	1·898	1·927	1·953	1·981	2·000
14·241	1·649	1·724	1·786	1·832	1·869	1·901	1·929	1·956	1·983	2·000
16·465	1·682	1·741	1·794	1·838	1·873	1·904	1·932	1·958	1·985	2·000
18·689	1·711	1·757	1·803	1·843	1·877	1·907	1·934	1·960	1·988	2·000
21·003	1·737	1·773	1·813	1·849	1·881	1·910	1·937	1·963	1·990	2·000
23·238	1·758	1·787	1·822	1·855	1·886	1·914	1·940	1·965	1·993	2·000
26·000	1·780	1·803	1·832	1·862	1·891	1·918	1·943	1·968	1·997	2·000
30·000	1·806	1·823	1·847	1·873	1·898	1·924	1·948	1·973	2·000	2·000
34·000	1·826	1·840	1·860	1·882	1·906	1·930	1·954	1·978	2·000	2·000
38·000	1·843	1·855	1·871	1·892	1·914	1·936	1·959	1·983	2·000	2·000
42·000	1·857	1·867	1·882	1·900	1·921	1·942	1·965	1·989	2·000	2·000
46·000	1·869	1·878	1·891	1·908	1·928	1·948	1·970	1·994	2·000	2·000
50·000	1·879	1·887	1·899	1·916	1·934	1·954	1·975	1·998	2·000	2·000
54·000	1·888	1·895	1·907	1·922	1·940	1·959	1·980	2·000	2·000	2·000

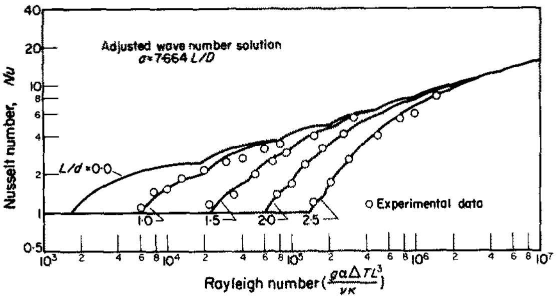

FIG. 2. Heat transfer in a cylinder with perfectly conducting walls.

the probable accuracy. Final assessment must be based upon experimental results. An observation that can be made is that the results here are exact in the vicinity of $\mathscr{R}_1$ when the exact value $\mathscr{R}_1$ and the accompanying velocity and temperature profiles are used. Thus the predictions have not only the correct value of Nu (unity) at $\mathscr{R} = \mathscr{R}_1$ but also the correct slope at $\mathscr{R}_1^+$.

If $\mathscr{R}_1$ is correct, if the slope of Nu vs. $\mathscr{R}$ correct in the vicinity of $\mathscr{R}_1$, and if the Nu vs. $\mathscr{R}$ relationship for $L/d = 0$ is nearly correct, then it is unlikely that $Nu(\mathscr{R}, L/d)$ could be greatly in error, for the $L/d = 0$ curve is essentially an asymptote for a finite L/d curve in the limit of large Rayleigh number.

The $\mathscr{R}_k$ computed are always based on a linear temperature gradient and recent attempts by Willis and Deardorf [14] to measure the $\mathscr{R}_k$ have shown poor agreement with Catton [6]. Somerscales [15], however, has recently collected a large number of data and attempted to obtain the transition points. He found no agreement among the values of $\mathscr{R}_k$ indicated by data of the various investigators. Conflicts existed even between sets of data taken by a single investigator. It appears that an extremely careful investigation must be made to discover true values of $\mathscr{R}_k$ for values of k larger than unity.

No attempt is made here to answer questions raised by Malkus and Veronis as to why a linearized analysis should yield nearly correct values of heat transfer for the case $L/d = 0$, but agreement between data and the Malkus–Veronis power integral theory is within 10 per cent [6] when the values of N_k and $\mathscr{R}_k$ computed as shown here are used in the prediction. This agreement then virtually guarantees reasonable agreement for all L/d as explained above.

Implicit in the method of analysis is an assumption that Prandtl number is not small. Use of the derived relations for Prandtl appreciably less than unity would not be warranted without further study.

It must be remembered in respect to practical application that, if the sidewalls are good conductors, the heat transfer through the walls may amount to as much or more than the transfer through the fluid; in this case the thermal resistance in the bond between the sidewalls and end plates may have a significant effect. In other circumstances, the walls may be poor conductors, but the fluid may be a gas transparent to infrared radiation so that elements of the sidewall have a net radiative heat flux which is transferred to the fluid. Such walls may behave more nearly like perfectly conducting walls than like adiabatic walls. Such effects have been neglected in the present treatment.

SUMMARY

Equation (24) gives Nusselt number vs. Rayleigh number as a sum of terms having critical Rayleigh numbers $\mathscr{R}_k$ determined by linear stability analysis and having power integrals N_k. Values of $\mathscr{R}_k$ may be estimated as shown by the procedure culminating in equation (35), and values of N_k have been calculated and tabulated in Table 1.

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Résumé—On emploie la technique intégrale en puissances de Malkus-Veronis pour prédire les densités de flux de chaleur dans des fluides contenus dans des cylindres droits verticaux chauffés sur leur fond horizontal et refroidis sur leur sommet. Des résultats sont obtenus pour des cellules avec des parois latérales soit isolées thermiquement soit conductrices. Des relations analytiques sont données pour le nombre de Nusselt en fonction du nombre de Rayleigh basé sur la hauteur de la cellule et sur un "nombre d'onde adapté" qui dépend du rapport hauteur sur diamètre pour les cylindres circulaires et des rapports hauteur sur côté pour les prismes à base carrée. Les prévisions sont comparées avec des résultats publiés auparavant.

Zusammenfassung—Mit Hilfe der Malkus-Veronis Potenzintegraltechnik wurde der Wärmestrom in senkrechten, flüssigkeitsgefüllten Zylindern berechnet, die am waagerechten unteren Ende beheizt und am waagerechten oberen Ende gekühlt sind. Die Ergebnisse wurden für Rohre mit adiabaten und leitenden Wänden erhalten. Die Ergebnisse werden in geschlossener Form angegeben für die Nusselt-Zahl als Funktion der Rayleigh-Zahl, die mit der Rohrhöhe und einer "angepassten Wellenzahl" gebildet ist, die vom Höhen- zum Durchmesserverhältnis für Kreiszylinder und vom Höhen- zum Seitenverhältnis für quadratische Zylinder abhängt. Ein Vergleich zwischen Berechnungen und kürzlich veröffentlichten Daten ist angegeben.

Аннотация—Интегральный метод Малкуса–Верониса используется для расчета теплообмена жидкостей в вертикальных цилиндрах, нагреваемых снизу и охлаждаемых сверху. Результаты получены для ячеек либо с теплоизолированными, либо с проводящими боковыми стенками. Приводится зависимость чисел Нуссельта от чисел Релея, построенных по высоте ячейки и по «волновому числу», которое зависит от отношения высоты к диаметру для круглых цилиндров и отношения высоты к ширине боковой стенки для квадратных цилиндров. Результаты расчетов сравниваются с данными, имеющимися в литературе.